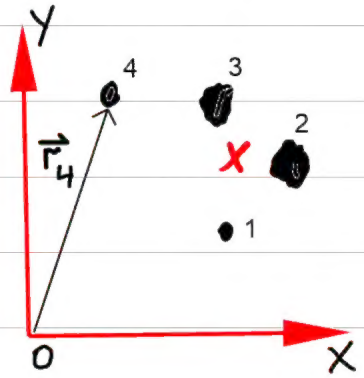
Meriam 4/1, 4/2: System of Particles – Newton's 2nd Law

A system consists of 4 particles, $i = 1,2,3,4$; $m = \sum_i m_i$

Define: $\bar{r}$ = center of mass

or $\vec{r}_G$ = center of gravity



$$\vec{r}_G = \sum_{i=1}^{4} m_i \vec{r}_i / m \quad \rightarrow \quad \vec{a}_G = \sum_{i=1}^{4} m_i \vec{a}_i / m \quad \star$$

Define:

$\vec{F}_i$ = sum of all external forces acting on m_i

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From Newton's 2nd Law, for each particle i,

$$\vec{F}_i + \vec{f}_i = m_i \vec{a}_i \quad , \; i = 1,2,3,4$$

$$\sum_{i=1}^{4} \rightarrow \quad \sum_{i=1}^{4} \vec{F}_i + \sum_{i=1}^{4} \vec{f}_i = \sum_{i=1}^{4} m_i \vec{a}_i$$

But from $\star$, this term equals $m \vec{a}_G$

→ For a system of N particles,

$$\sum_{i=1}^{N} \vec{F}_i = m \vec{a}_G$$

(only external forces count)

Meriam 4/1-4/3 : System of Particles — Work & Energy

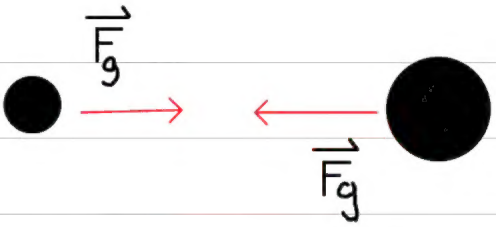
for each particle i, $\left[(T_2)_i = (T_1)_i + (U_{1\to 2})_i\right]$ ★

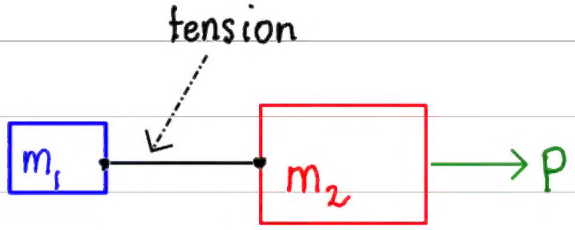
Sum eqn ★ for all particles...

$$\rightarrow \boxed{\sum_{i=1}^{N}(T_2)_i = \sum_{i=1}^{N}(T_1)_i + \sum_{i=1}^{N}(U_{1\to 2})_i}$$

Should we include internal work performed by one particle on another?

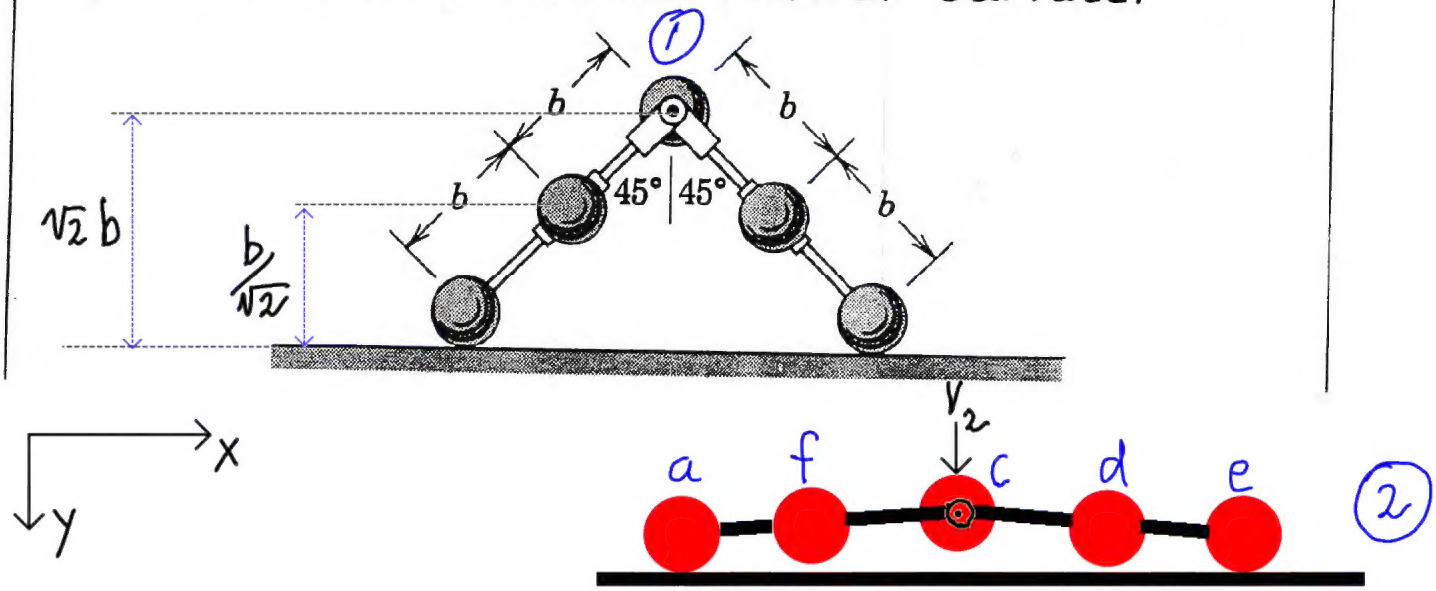
eg.





WORK-ENERGY

The system of five identical spheres is released from rest in the position shown and moves in the vertical plane under the influence of gravity. The two rigid arms are freely hinged to the upper sphere. Neglect friction and find the velocity v_2 with which the upper sphere strikes the horizontal surface.



As spheres approach final position ②, $\vec{V}_a = \vec{V}_e = 0$

(at position ②, spheres a, e are centers of zero velocity).

Also, $V_f = V_2/2$, $V_d = V_2/2$ at position ②

$$T_2 = T_1 + U_{1\to 2}$$

(T_1: 0; $U_{1\to 2}$: $-mg\Delta h$)

f_2 c_2 d_2 f_g c_g d_g

$$\tfrac{1}{2} m\left(\frac{V_2}{2}\right)^2 + \tfrac{1}{2} m V_2^2 + \tfrac{1}{2} m\left(\frac{V_2}{2}\right)^2 = mg\, b/\sqrt{2} + mg\, b\sqrt{2} + mg\, b/\sqrt{2}$$

$$\rightarrow V_2 \text{ for center sphere} = 2\sqrt{2\sqrt{2}\, gb/3}$$

Meriam 4/4: Impulse-Momentum for System of Particles

System of N particles with center of mass G at $\vec{r}_G$

Let $\vec{\rho}_i$ = position of particle i wrt G

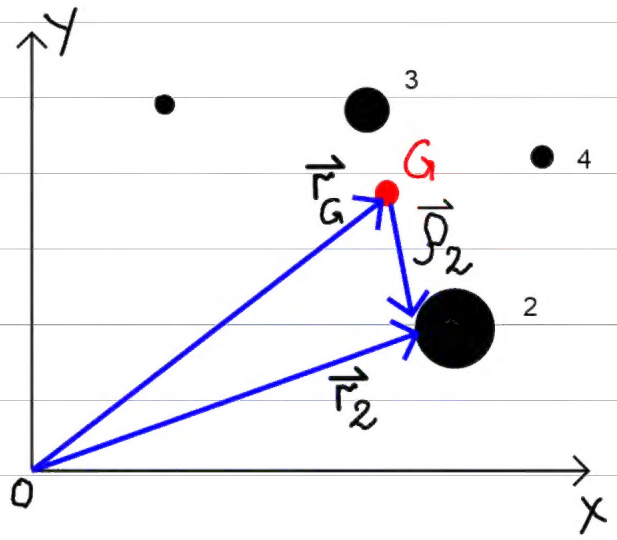
$$\rightarrow \quad \vec{r}_i = \vec{r}_G + \vec{\rho}_i$$

$$\rightarrow \quad \vec{v}_i = \vec{v}_G + \dot{\vec{\rho}}_i$$

$$\vec{r}_G = \frac{\sum m_i \vec{r}_i}{m} = \frac{\sum m_i (\vec{r}_G + \vec{\rho}_i)}{m}$$

$$\rightarrow \vec{r}_G = \frac{\sum m_i}{m} \vec{r}_G + \frac{\sum m_i \vec{\rho}_i}{m}$$

($\frac{\sum m_i}{m} \rightarrow 1$, $\frac{\sum m_i \vec{\rho}_i}{m} \rightarrow 0$)



For the entire system of N particles,

$$\vec{G} = \sum_{i=1}^{N} m_i \vec{v}_i = \sum_{i=}^{N} m_i (\vec{v}_G + \dot{\vec{\rho}}_i)$$

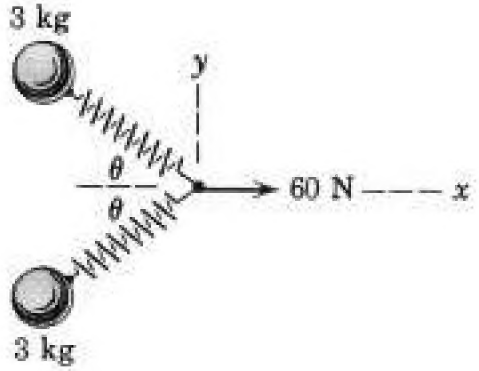
$$= \sum_{i=1}^{N} m_i \vec{v}_G + \frac{d}{dt}\left[\sum_{i=1}^{N} m_i \vec{\rho}_i\right]$$

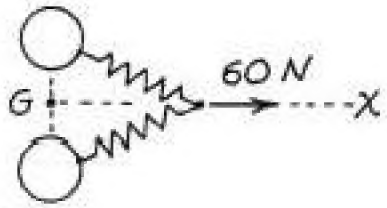
$$= \vec{v}_G \sum_{i=1}^{N} m_i + \frac{d}{dt}[0]$$

$$\boxed{\vec{G} = m \vec{v}_G} \quad \rightarrow \quad \dot{\vec{G}} = m \vec{a}_G = \sum \vec{F}$$

$$\rightarrow \quad \boxed{\vec{G}_2 = \vec{G}_1 + \int_{t_1}^{t_2} \sum \vec{F}_i \, dt}$$

The two 3-kg spheres slide on a smooth horizontal surface in the x-y plane and are connected by the two identical springs of negligible mass. Calculate the x-component of the acceleration of each sphere due to the action of the 60-N force applied to the junction of the springs.





Center of mass G of the system is always midway between spheres, so x-component of accel. of each sphere equals $\bar{a}$

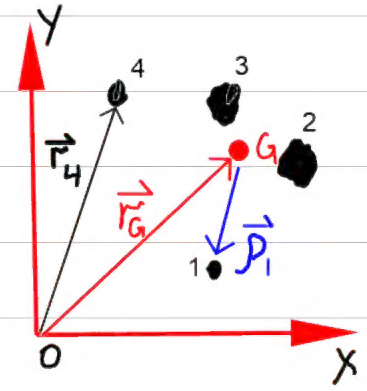
For system $\Sigma F_x = m\bar{a}_x$, $60 = 2(3)\bar{a}_x$

$\bar{a}_x =$ $\boxed{10\ m/s^2}$

JLM

Summary

For a system of N particles, e.g. $N=4$,



- $\vec{r}_G = \sum_{i=1}^{4} m_i \vec{r}_i \,/ m$
- $\sum_{i=1}^{N} \vec{F}_i = m \vec{a}_G$ (only external forces count)
- $\vec{G} = \Sigma \vec{G}_i = \sum_{i=1}^{N} m_i \vec{v}_i = \sum_{i=1}^{N} m_i (\vec{v}_G + \dot{\vec{\rho}}_i)$

→

- $\sum_{i=1}^{N} (\vec{G}_2)_i = \sum_{i=1}^{N} (\vec{G}_1)_i + \int_{t_1}^{t_2} \Sigma \vec{F}_i \, dt$
- $\sum_{i=1}^{N} (T_2)_i = \sum_{i=1}^{N} (T_1)_i + \sum_{i=1}^{N} (U_{1\to 2})_i$

Next Lecture

- Read 4/4 – 4/5 (Energy & Momentum)